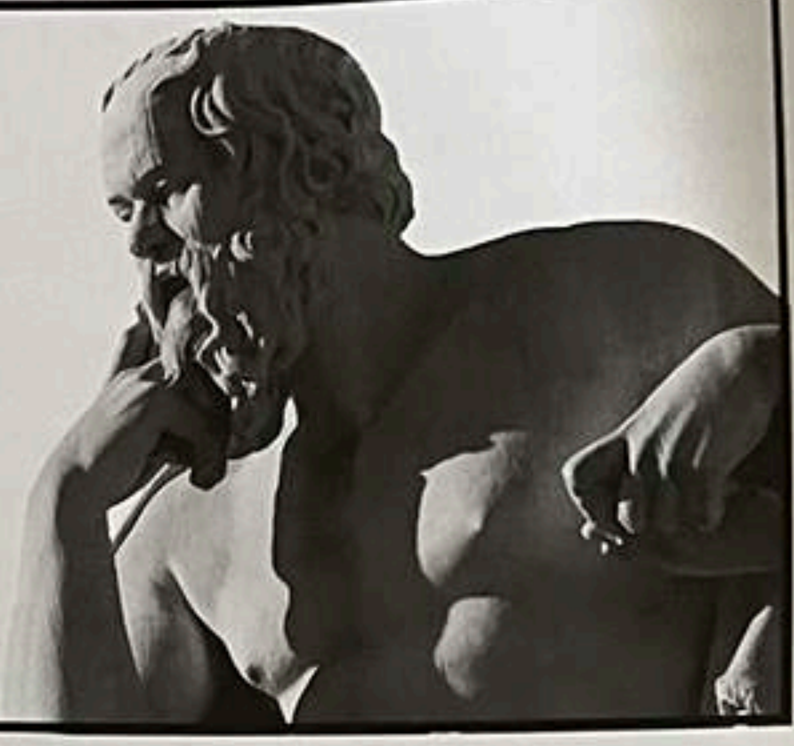
PHILOSOPHY AND HISTORY AS AN EPIC NARRATIVE IN SECONDARY SCHOOL MATHEMATICS



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Introduction

In this article we propose an approach to the incorporation of historical and philosophical foundations of mathematics in the teaching of school mathematics. The use of philosophical enquiry and study has already been made rather successfully for younger children by the Philosophy for Children (P4C) program. We will begin by briefly summarising P4C and acknowledging evidence that supports inclusion of philosophical topics and enquiry generally. We will then explore the question of what form these foundational features should take, specific to mathematics education. Building on a foundation inspired by P4C we strive to broaden the role that historical and philosophical foundations can play in education, and from that enlarged perspective make an argument for specific purposes, and associated methods.

Philosophy In The Classroom

One of the endearing characteristics of traditional and indigenous cultures is the fact that life skills are learned in the context of ongoing communal life whereby lessons are presented in context with the artful uses of narrative and story-telling. These are naturalistic approaches and were sufficient to transmit the entire cultural heritage of the social group to the next generation.

With the long progression toward modern living, all that changes. Education becomes sequestered, and much of it is conducted in ways that dissociate subject matter from the ambient culture, and at times this means the curriculum dissociates from the intellectual disciplines it represents, so that teaching, learning, curriculum design, and assessment become a self-contained, other-worldly pursuit operating in its own existential universe. Students 'go to school' and 'study subjects,' but the sense of participating in the larger social and cultural enterprise has been severely compromised, so that students are left to wonder how any of this is 'relevant,' or 'meaningful,' or even useful. Education is what takes place in schools divorced from the world 'out there', yet it is perfectly reasonable to search for ways of reconstituting some of the crucial connections and conceptual richness between the curriculum and intellectual culture that have been severed by the isolating purification and compartmentalization of subject matter.

The curriculum must not simply package 'knowledge and skills' but should serve to constitute an educated mind, able to survey and comprehend a full view of the entire culture. That requires more attention paid to achieving deep semantic understanding of each domain of learning, to cultivating a much richer fabric of connections between disciplines, and between the learner's mind and the complex intellectual culture for which the curriculum serves as proxy.

Despite the dedication of the teachers, the profession struggles to find the combination of knowledge, skills, dispositions, curricular coherence, and pedagogical mastery that would result in every learner achieving their full intellectual potential. We struggle with the effects of an educational system founded in the image of large industrial organizations, where mass production and specialization became the key to manufacturing efficiency. We continue on with curriculum that has been analysed, dis-integrated, and arranged for piece-by-piece assembly, without adequately conveying to students or teachers the gestalt-like structure, clarity, conceptual integrity or intellectual beauty of these domains of knowledge (see Senechal, 2014).

Mathematics is inherently abstract, symbol-based and self-supported by its axioms, yet it is also the key subject area for comprehending the cognitive and cultural underpinnings of modern civilizations. Its complexity needs to be clearly and deliberately revealed. An effective way to do this is by tracing the genealogy of concepts as they evolved from naturalistic origins into the highly abstract and conventionalized systems of knowledge of today. Although this would require a systemic rethinking of the entire curriculum, our preliminary recommendations may be tested by any teacher of mathematics seeking to gain better clarity for students in the underlying nature and structure of their discipline.

The teaching profession has entered a phase of reform and revision characterized by data-driven decision-making and accountability schemes to define and align all that goes on in schools. None of this will fundamentally change a curriculum viewed by many students as essentially incoherent, atomized, disconnected, incomprehensible, irrelevant, and meaningless – each subject sealed off from the others, and the whole thing a vague proxy for the larger intellectual culture out there in the real world. It is a perverse form of motivation that substitutes extrinsic rewards for the deeper intrinsic and natural motivations, which in themselves, alas, need to be cultivated assiduously in order to become activated for most learners.

Every subject (every *intellectual discipline*) has an array of features which are properly characterized as philosophical in nature. This is where the ontological and epistemic nature of the subject is defined. It is a venerable but neglected notion that the history and philosophy of a given intellectual discipline adds value to students' ability to understand that subject deeply and clearly, and to understand its role within the larger cultural scheme of things. These are 'foundational perspectives,' which tell us much about how these intellectual disciplines figure into the cultural and social life of a community, a society, or humankind in general.

Philosophy For Children

There are pedagogical programs that serve to incorporate foundational perspectives into teaching. A prominent example worldwide is *Philosophy for Children* (P4C), created by Matthew Lipman during the late seventies.

Most of the current articles on philosophy in the mathematics classroom are influenced by the P4C movement and the philosophical inquiry of the class is structured by an emphasized Deweyan democratic philosophy (Lipman *et al.* 1980) in which it is the class that takes the lead in the discussion. According to Haynes:

> Central to the practice of philosophy with children is that all discussion arises from children's questions, usually in response to a particular stimulus, such as a story, picture or poem. By first examining all the questions the discussion is able to gravitate towards those questions that are open-ended and have no obvious answer. The process of choosing a question and of engaging in enquiry is a democratic one in which the adult in charge strives to enable the children's discussion to follow its own course, rather than steering it towards a planned goal. (Haynes, 2012, p.12).

P4C's intriguing characteristics have to do primarily with the socially democratic nature of learners' respect, appreciation, and acceptance of one another, and for the elegant manner in which conversation carries forward on the basis of a group dynamic, rather than on the authority of the teacher or dominant student. P4C also cultivates a habit of meta-cognition and mental attitudes that facilitate clear reasoning, open-mindedness, ability to consider from different perspectives, objectivity and so forth. Disadvantaged learners in particular are often disabled by the belief that they are not capable of these kinds of cognitive skills. Overcoming such pernicious beliefs in learners may be a subliminal effect of the P4C focus on democratic social conditions defining the community of enquiry.

It is an important part of P4C that topics *not* be defined by the teacher, because P4C is not about teaching philosophical topics, it is about teaching philosophical methods of discourse and enquiry, which must be cultivated within a dialectical community. At some point, it becomes possible to build upon that foundation of cognitive and social skills by introducing specific topics, and then employing the community of discourse to unpack the complexity and relevance of those topics. This suggests a judicious use of direct instruction, because in such cases there are indeed specific topics to be learned. Pleasingly, though, the fact that P4C has given respectability to the practice of philosophical enquiry can help to establish the important point that the teaching of philosophical topics requires active forms of learning, especially when invested within a discourse community.

What Is The Modern Equivalent?

We do not propose taking an exclusively non-directive approach that allows the conversation to develop organically in whatever direction the group culture takes it, yet there are compelling aspects of the P4C approach that are foundational for any community of enquiry: A social contract that allows, encourages, respects, and supports students so they are willing to cross the threshold of protective silence to share their thinking with others, thus to cultivate a group culture that fosters curiosity and embodies a method of shared discourse conducive to the kinds of conversations Haynes (2012) and others have described occurring because of the P4C framework. The cultivation of such attributes should begin with young children, because those methods serve universally to create the non-threatening, open-minded environment needed just to get the habit of philosophical reflection and metacognition started (see Note). The alternative is painfully familiar - the class moves on and builds new knowledge upon a foundation some students don't comprehend.

Goals And Purposes

Students do not need foundational perspectives to engage in tasks of rote learning or applying rule of thumb procedures for examination success. It is when we turn to matters of comprehension – deep semantic understanding – that philosophy comes into play, because it serves to map out the territory of possibility from which a particular configuration of concepts and conventions was chosen to define some critical aspect of the discipline. There is always a creative, artistic element in the development of mathematics, or science (or any other domain of knowledge), which integrates in complex ways with that knowledge given by empirical research, logic, and prior disciplinary traditions.

Disentangling these very different types of influence is almost impossible without the kind of background we are suggesting, but without this effort to understand the nature of knowledge the learning must simply be taken on faith. That is tantamount to an ultimatum, which often serves to break the circuit between meaning, interest, and motivation. So, while students may not require philosophy to engage in the inevitable but unstimulating labour of applying rule of thumb procedures, repetitive practice etc., that work can become far more palatable when it can be seen in the service of larger pedagogical and cultural aspirations the student has acquired by experiencing the intrinsic beauty and fascination of the discipline. Seeing how the subject was built up from its naturalistic origins through cultural-historical processes of development is arguably the best way to give students access to those intuitions and understandings. This requires some minimal engagement with historical and philosophical foundations.

The subject as taught in schools embodies the consequences of those cultural-historical events, decisions, and discoveries, but our custom has been to present them as *fait accompli*, a status that for students is practically indistinguishable from a pretension of absolute Truth. To announce, as Euclid did in the beginning of The Elements, that "A line is breadth-less length" is to pack an awful lot of presumption into a seemingly innocent but incomprehensible statement - incomprehensible because human imagination cannot conceive of a line that has no width, since the "image" ceases to be visible once the width shrinks to zero. Now the student is left with a dilemma, either to accept on faith something that violates all prior experience and common sense, or to simply refuse to proceed further into a territory predicated on what appear to be patently obvious falsehoods. We routinely treat this and other cognitive crises in a cavalier and dismissive manner, expecting learners to violate their own internal warning system and go along with what we say, simply based on teacher authority. Why is this important? Because intellectual confidence should be built up on the basis of evidence and reason, not on a system of authoritative fiat, whenever possible.

The system we are going to share has been tested informally on 11- and 12-year-old students as an excursion into the foundations of geometry in the context of a social studies class where the semester topic was the history of ancient Greece. As this was not a formal study, no canonical conclusions should be drawn from it, but the experience reflected precisely the high degree of student engagement, curiosity, excitement, and thoughtful patterns of response characterized by Haynes in her writings. So, we are reasonably confident in suggesting that this is a direction that appears promising and should be considered by other researchers as a means of engaging students in a quest for foundational understanding of the disciplines they are learning about in school. Doing so taps into sources of surprisingly deep curiosity and desire for learning, even amongst the most recalcitrant and jaded of learners. We were continuously left with the distinct impression that this is exactly what students are hungry for.

These middle school students had previously studied early civilizations, so it was already familiar territory when the unit on Classical Greece made reference to Egypt, a place that Thales visited. One of the authors of this paper served as an invited guest over the course of several weeks, and this is how he began the story:

> *It is late. The sun has gone down, calming the intense heat of the Egyptian desert. Thales looks up into the night sky and sees three stars. His mind connects the three stars with invisible lines. For just an instant he believes that he can see those lines too, just as clearly as he sees the stars. 'Those three stars have formed a triangle,' he says to himself. He blinks. The image fades. The stars still shine brightly, but the lines he can no longer see. 'What of those lines?' he wonders. 'I know the stars are real; but weren't those lines real too?' He smiles. He answers his own question, saying quietly to himself, 'No. The stars are real, for the evidence of them persists. But the lines I saw did spring from my mind, and they now have faded from imagination. They were only ideas.' But what about ideas? Aren't they real too?*

The first few lines of this brief narrative position the listener in a different time and place. It is night, it is Egypt, it is hot, it is a desert place, and we are looking at the night sky through the eyes of Thales. We even travel inside of his imagination to participate in a private conversation, one that we ourselves might have experienced under the same circumstances. This story, pronounced carefully, and enacted with some demonstrative gestures, took just over one minute, but in that time all of the students were willingly transported into a moment in the intellectual life of our protagonist, Thales. Discussion ensued, with great enthusiasm over this wonderful little question, "Aren't ideas real also?" This is an ontological question, asking in what sense ideas can be said to exist. This lays a foundation for the discussion that will occur later about the 'reality' of theoretical objects, such as points, lines, triangles, and circles, and for Plato's astonishing answer to that question (that these 'Ideas,' which he called "Forms," are more real, even, than concrete objects, because they embody properties that are logically necessary, hence immortal, immutable, and therefore absolute).

The following day, this same guest took the students back to Egypt again, this time to listen in on a conversation between Thales and those Egyptian priests who knew the secrets of geometry. In that instance, they are examining a configuration of four wooden stakes, pounded into the ground, with two ropes stretched tightly between opposing stakes to form an elongated "X". Unlike the configuration of perpendicular lines, adjacent angles are no longer equal, and no longer right angles, yet the older Egyptian priest has pointed out that opposite angles are equal. And Thales has agreed, saying, "Yes, that is obvious... But, how would you *prove* it?" The Egyptians are silent for a moment, then burst out in laughter, and finally the older fellow asks, "Why on earth would you need to prove something when it is so obvious?"

In that moment, with those sixth grade (year seven) students, fully engaged in this powerful little narrative, we have replicated a pedagogical circumstance familiar to anyone who has read Haynes' books, the point where the problem space has been identified, and we then turn to the students and ask them to formulate their own questions, which can then become the basis of whatever discourse will ensue. The teacher at this point is faced with tactical decisions as to how much the locus of control shall disperse outward to the students, and for how long. The teacher must also find a way to share with the students how the circumstance portrayed in this legend played out. (We identify the story as a *legend* rather than as a *history*, since no one knows exactly what transpired between Thales and the Egyptian geometers, only that, according to Aristotle and Proclus, he spent twelve years in Egypt and was said to have befriended the priests responsible for a knowledge of geometry).

In the classroom event described above, students came up with several very good questions, including "Right, why would you want to *prove* it if you already *know* the answer?" and "Is it *really* obvious? I am not sure." And "Yeah, that *is* a good question. How *could* you prove it?" This last question was particularly prescient, since the point of this story was to introduce the fact that Thales puzzled deeply over the question of what would constitute an acceptable form of demonstration, which helps to answer a mystery: Why does posterity credit Thales with proving the above proposition, when the thing itself is frankly self-evident?

With this line of inquiry, we have moved this group of students into a conceptual problem space, the 'point of inflection' in the course of mathematical history from which the concept and practice of 'formal proofs', based on logic rather than measurement or calculation, is born. In a subsequent story, Thales will pass on his interest in geometry to Pythagoras, who actually *did* develop the beginnings of formal proof, and achieved the transfiguration of geometric figures from concrete objects acted upon by empirical measurement, to theoretical objects acted upon by means of pure reason. It is in that transcendence that a 'line' sheds its width, and the conversation in geometry becomes one of logic rather than measurement.

A love of knowledge is an acquired taste, of course, but it helps if we can understand how to appreciate it. In the above narratives, we are experiencing (vicariously) the beginning of a social process that created classical geometry. And because it is being presented in this narrative manner, each student gets a seat at the table, from which it becomes possible to see how the beginnings of formal proof excited interest in those who make some of the earliest discoveries. In the process, they learned a style of reasoning they called 'rationality.' In other words, they deliberately cultivated a fundamental property of the modern scientific mind, the ability to reason sequentially and logically from premises to conclusions, and to make those thought processes visible to inspection and open to critique.

In our work, we have defined these historical 'points of inflection' as "primary transformative events." In Carson and Rowlands (2007) we proposed that a total of just 17 of these conceptual or procedural transformations would account for the historical course of Euclidean geometry. So, there is a basis for parsimony in the use of these foundational perspectives, namely a targeted focus on the events in which there is a significant transformation in "the co-evolution of culture and cognition." A P4C approach does come into the method of teaching, but the locus of control returns to the teacher, who then explains how that particular problem space was resolved historically. This is sufficient to allow students to enter into the relevant problem space and to follow the course of each transformative event, thus to understand upon what basis the advancing state of the mathematical discourse is proceeding.

Defying tradition, we would not start with Euclid but with the Egyptians and Thales. In other words, start at the beginning of the cultural-historical sequence, and allow the students to follow the developmental progression of concepts, the evolving transfiguration of the discipline, vicariously, as if they themselves are present in those critical moments in which the major revolutions of new concepts, new discoveries, new intellectual conventions, and new procedures occur. We do not have to know the exact history (for often we do not), but what we do need is to encapsulate what we do know into a viable historical fiction that embodies the essence of each respective transformational event, so that the problem space and its resolution can be experienced viscerally as well as intellectually by students. We do so by prefacing: "Let's imagine how this event might have occurred..." or something to that effect. It is intended as an artful simulation but based on as much factual knowledge as possible.

Conclusion

Much of school mathematics is abstract, and although many learners know how to correctly apply the rule of thumb procedures to arrive at correct solutions, we fail to help them understand the relations between the central concepts that make up the topic, and to grasp qualitatively those concepts. However, the context is ever present to make the abstract the focus for discussion. For example, what is a geometric straight line? How does one know that the three angles of a (plane) triangle add up to two right-angles? Is deduction (such as the proof of the angle property of the triangle) better than induction (such as measuring the angles of a number of triangles)? Can velocity be measured at an instant? Can we find the exact number of unit squares that fill the area under a curve? What is the next number after zero? Etc. (see Rowlands, 2014).

Although philosophy as a bolt-on can, by itself, have a beneficial effect in understanding the mathematics, some well-crafted narrative history not only contextualises the difficulties faced by the learner but together with the relevant philosophy can also highlight the cultural significance of concepts under study. Greek geometry is a good example of a theoretical practice that has permeated society and culture (Alexander, 2019), and as difficult as it may be to gain a mastery, it may also be the single best opportunity most students will ever have in understanding the uniqueness of scientific culture as a human achievement, since its cognitive foundations were originally developed among the philosophers and mathematicians of ancient Greece.

This is not history necessarily in terms of understanding precisely how our ancestors saw the problem and their ways to solving it, since in many cases we do not have direct historical knowledge. Nor are we unsympathetic to the various concerns with equity (e.g. Fried 2014) that a history of knowledge becomes a Whiggish celebration of 'progress' - or a grand narrative that overshadows other cultural narratives. It is really story telling we are after, informed by historical knowledge. More to the point, we seek to humanize mathematics for students by placing its development within an accessible human context, namely the story (the 'epic narrative' of its cultural-evolutionary development). Without this human and cultural context, mathematics is too often experienced by learners as an unending series of facts and procedures to be memorized, for no apparent reason.

Story-telling is powerful (as every indigenous community and ancient society knew) because it situates the listener's imagination within the personae of the story's protagonist, thereby simulating the *felt experience* and *embodied cognition* (D'Amasio, 1999) of those narrative figures. This brings to life the struggles, but also the resulting epiphanies, discoveries, moments of innovation, and development of mathematical conventions that served to create this discipline. From that vantage point, students can experience those same cultural and cognitive revolutions vicariously. Philosophical guidance by the teacher can then shape the resulting classroom discussion around the culturally and conceptually relevant topics and provide those topics with suitable terminology. Well-designed curricular materials can help.

We don't know the development of Thales' proofs that culminated in the protocols of deductive proof established by the Pythagoreans, but we can be honest and say we don't know, inviting the class to think of possibilities of how this came to be. We want them to be curious about such matters.

Note

This article is predicated on the existence of the dialogical community of inquiry's practice; but if such a practice does not exist in the school and the catchment area, then one may be created and hopefully timetabled as it is in many primary/junior schools in the UK. Sowey's (2021) *Unveiling and Packaging* explains the protocols for democratic inquiry and the tools to ignite the passions, to induce perplexity, to challenge intuitions, to dispassionately reflect on the arguments of others including one's own and to illicit reasoned argument. This is an essential article for any teacher who would like to set up a community of inquiry, either as a timetabled activity, an activity with the form teacher or as an afterschool club.

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